Pixel-Exact Vector Reconstruction and Joint Irreducibility Certification

Korean Patent Application No. 10-2026-0208000 (pending) November 2026

Zero tolerance
Every pixel matches after rendering and quantization

Per joint
Irreducibility certificate issued for each joint of the path

Exact arithmetic
Every rejection confirmed in exact rational arithmetic

Verifiable
Certificates re-checkable by a separate implementation

1 Overview

The technology reconstructs a cubic Bézier path from a quantized raster image. When the reconstructed path is rendered and quantized with the given rendering model, every pixel value matches the input image.

For each joint of the path, it establishes by computation that no single cubic curve within a bounded range can replace the two adjacent segments and still reproduce the input image, and outputs this result as a certificate. The certificate can be re-checked by an implementation separate from the one that produced it.

2 Inputs and Outputs

Type	Item	Description
Input	**Quantized raster image**	8-bit pixel image, stored losslessly.
Input	**Rendering model**	Rules for computing the per-pixel coverage of a curve path, and the quantization rule.
Output	**Witness path**	Sequence of cubic Bézier segments connected at joints. Re-rendering matches every pixel.
Output	**Per-joint certificate**	Partition of the candidate parameter region and the rejection basis for each partition cell.
Output	**Verification record**	Results of the re-rendering comparison and the certificate re-check.

3 Processing Steps

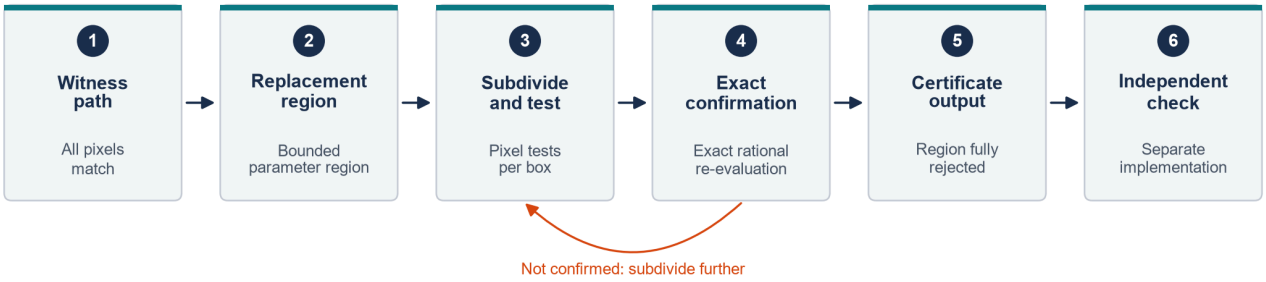


Figure 1. Processing steps

1. **Witness path.** Obtain a cubic curve path whose rendered and quantized result matches the input image at every pixel.
2. **Replacement region.** For each joint, define the parameter region of candidates that replace the two adjacent segments with a single cubic curve whose outer endpoints are fixed. The region is bounded by requiring the curve to stay inside the image area extended by a set margin (Figure 2).
3. **Subdivide and test.** Recursively subdivide the parameter region into boxes. A box is rejected when every replacement curve in it fails to reproduce the observed value at one or more pixels.
4. **Exact confirmation.** Floating-point computation is used only to select rejection candidates. A rejection is confirmed only by re-evaluation in exact rational arithmetic; otherwise the box is subdivided further.
5. **Certificate output.** When rejected boxes cover the entire parameter region, an irreducibility certificate is issued for that joint.
6. **Independent check.** Re-check in exact rational arithmetic that the path re-renders identically, that the boxes partition the region with no overlap or gap, and that the rejection basis of each box holds.

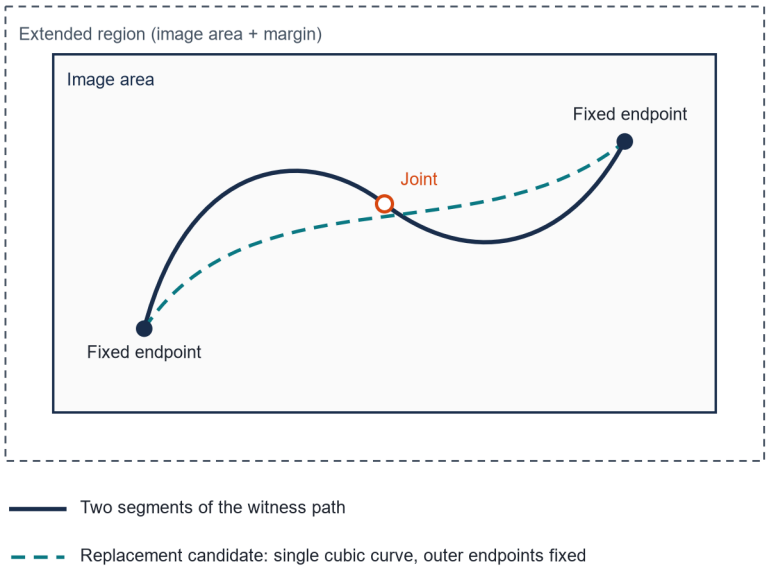


Figure 2. Joint replacement

4 Decision Criteria

Item	Criterion
Pixel match	All pixel values identical after rendering and quantization. No tolerance.
Joint irreducibility	No candidate that replaces the two segments at a joint with a single cubic curve, outer endpoints fixed, satisfies the pixel match within the bounded parameter region.
Certification unit	Per joint (local).
Arithmetic	Rejections confirmed in exact rational arithmetic. Floating point used only to screen candidates.

5 Operating Conditions

- Images produced under a defined rendering rule. The coverage and quantization rules of the target program are built into the rendering model.
- Losslessly stored images. For lossy-compressed images, the compression step must be modeled.
- Scanned or photographed images require an added observation model for blur and noise.
- Two-dimensional images and curve paths. Three-dimensional shapes are out of scope.

6 Evaluation Results (Internal)

Item	Result
Sensitivity of the pixel-match criterion	In test images, moving one control point by 0.5 pixel changed 15 to 36 pixels.
Against tolerance-based simplification	Irreducibility was certified for most joints that 1-pixel-tolerance simplification removes. Removing those joints changes pixels of the original.
Certificate re-check	Certificates re-checked box by box with a separate implementation. 0 mismatches.
Re-rendering comparison	See Figure 3.

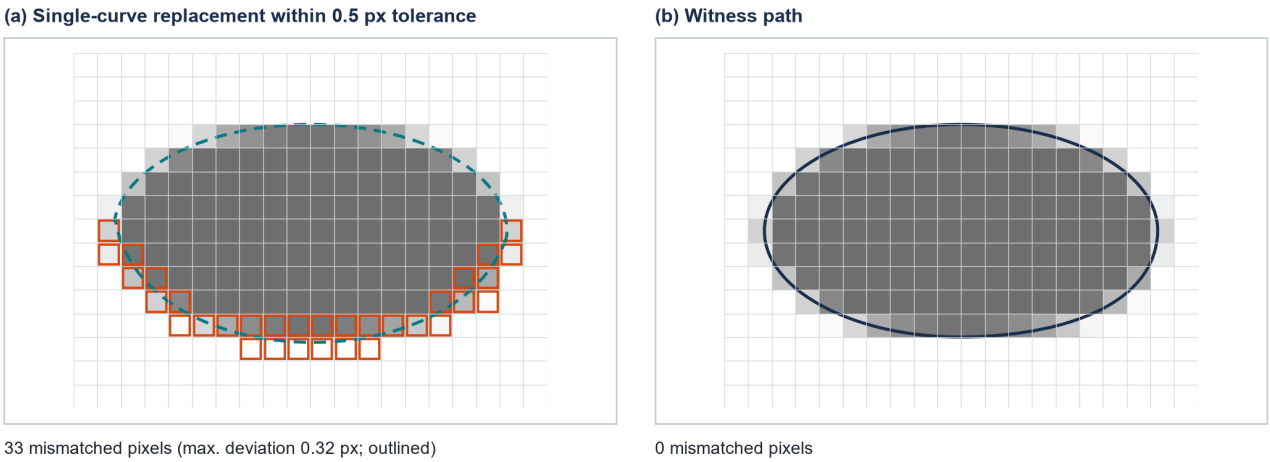


Figure 3. Re-rendering comparison: single-curve replacement within 0.5 px tolerance (a) and witness path (b)

7 Comparison with Conventional Methods

Item	Tolerance-based methods	This technology
Decision criterion	Geometric distance or loss-value tolerance	Every pixel identical after rendering and quantization
Result check	Error figures, visual comparison	Pixel-by-pixel re-rendering comparison
Basis for simplification limit	None	Per-joint irreducibility certificate
Arithmetic	Floating point	Exact rational arithmetic
Third-party verification	Rerunning the same program	Separate implementation re-checks the certificate

8 Applications

Field	Application
Archaeology and cultural heritage	Vector reconstruction of artifact and site survey drawings, pottery cross-sections and pattern unrollings, with verification against the original. Vector records of drawings output from 3D scanning and photogrammetry, digital rubbings and virtual restorations. Outline records of characters and patterns in rubbings and inscriptions.
Archives and historical documents	Reconstruction of figure and character outlines from digitized old maps, books and documents. Comparison of restored historical typefaces and movable type against the original.
Forensics and digital examination	Pixel-level comparison of seals, logos, figures and character images in electronic documents against reference originals. Preparation of reference material for seal impression comparison.
Fonts	Glyph outline reconstruction, rendering comparison, and verification that rendering is unchanged after point reduction.
Digital records	Vector reconstruction and match verification of 3D and CAD output drawings and of images in preservation formats.
Vectorization software	Verification of conversion results; per-joint check that no two segments can be merged into one.
Scanned and photographed material	Rubbings, survey drawings, old maps and paper seal impressions, after the observation model is extended.

9 Intellectual Property and Contact

Patent status	Korean Patent Application No. 10-2026-0208000 (pending)
Title	Method and System for Certifying Local Irreducibility of a Vector Path that Exactly Reproduces a Quantized Raster Image
Trial application	Prototype evaluation available to confirm actual performance before full deployment or a transaction.
Contact	Tel +82-10-8499-2397 | Email khj243@primarysystem.kr | Web primarysystem.kr

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